

percent at X band with insertion loss less than 0.5 dB over 79 percent of that bandwidth. Massé [6] described a TT1-390 mechanically inserted puck-type X-band device which gave a 32.5 percent 20-dB bandwidth with insertion loss less than 0.5 dB over the band.

A portion of the excess insertion loss of these APS circulators is due to the length of the microstrip lines required in the packaging. However, some of the loss is likely due to a slight erosion noted at the ferrite-dielectric peripheral interface after lapping. This is caused by the thermal-expansion mismatch between the D-13 dielectric and the ferrite which results in the ferrite shrinking away from the peripheral wall, or the creation of tensile stress in the two materials at the interface. Subsequent lapping removes the stressed material in this region at a faster rate than the rest of the surface, resulting in a slight trenching effect. This caused the final chrome-gold metallization to be degraded slightly in quality in the trenched region. This effect was most pronounced in TT1-105 compared to other material systems examined and may be alleviated through appropriate selection of ferrite and substrate materials.

It is felt that the preliminary results reported here demonstrate that APS material is well suited for inserted puck-type circulators, and that with optimized layout and fabrication procedures, the performance of APS circulators can be at least as good as solid-substrate circulator designs. One advantage of the puck-type circulator is that the transformers are fabricated on a dielectric substrate rather than on ferrite and so can be accurately designed. The fringing fields from the bias magnets will have no effect on the transformer behavior in this case.

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## A Millimeter-Wave Reflection-Beam Isolator

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**Abstract**—A new and simple type of millimeter-wave isolator using a solid-state magnetoplasma in a reflection-beam system is described. Some data are presented showing performance at 94 GHz. Practical considerations indicate that performance should be much closer to ideal at higher frequencies.

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## I. INTRODUCTION

We present here a solid-state magnetoplasma isolator with simple configuration for use in a millimeter-wave reflection-beam system. The geometry of this nonreciprocal mirror is that of the Kerr transverse magnetooptic effect, wherein electromagnetic (EM) waves propagate perpendicular to a dc magnetic field and are reflected from the surface of the plasma, the wave polarized in the plane of incidence. Large nonreciprocal behavior with this geometry was explained for EM waves reflected from the ionosphere by Barber and Crombie [1].

The same configuration with the gaseous plasma replaced by a semiconductor, GaAs, has been studied in both theory and experiment for a wide range of material parameters. But the large nonreciprocity observed for the ionosphere was not found, due to the relatively large background dielectric constant  $K_L$  of the semiconductor lattice. In this short paper we extend the calculations of Wait [2] and Seaman [3] to include not only the permittivity of the semiconductor lattice, but also a dielectric half-space in which the incident and reflected waves travel. The inclusion of such a dielectric markedly improves the degree of nonreciprocity and efficiency of the device as an isolator. We also show experimental behavior of a small isolator at 94 GHz and room temperature. This short paper presents results more complete and more nearly ideal than the preliminary report of [4].

## II. THEORY

A convenient means for dealing with EM waves in a solid-state magnetoplasma is to characterize the medium as a complex dielectric tensor which we calculated for the simplest case with isotropic electron effective mass  $m^*$  and isotropic energy-independent collision time  $\tau$ . Anisotropy of the conductivity arises from the presence of an applied dc magnetic field  $B_0$  which causes the charged particles to orbit at the cyclotron frequency  $\omega_c$ .

The reflection coefficient  $R_M$  for a plane boundary between a semi-infinite dielectric medium and a semi-infinite plasma can be shown to be [4]

$$R_M = \frac{\cos \Theta - \Delta_M}{\cos \Theta + \Delta_M} \quad (1)$$

where  $\Delta_M$  is a function of the angle of incidence  $\Theta$  (shown in Fig. 1), the semiconductor parameters  $m^*$ ,  $\tau$ ,  $\omega_c$ , and the dielectric constant  $K_M$  of the semi-infinite dielectric medium. When collisions are present ( $\tau$  is finite, neither zero nor infinite), the reflection coefficient  $R_M$

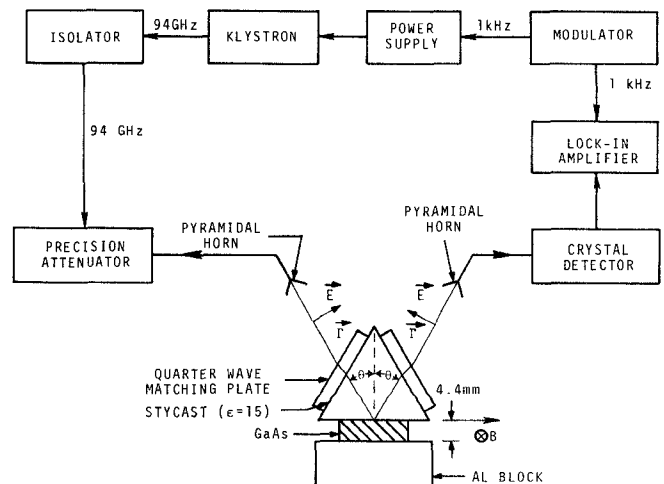


Fig. 1. Experimental setup used to measure reflection from GaAs at 94 GHz.

is found to be nonreciprocal; that is, reversal of the direction of propagation, or alternatively, the sense of the dc magnetic field, changes the magnitude of the reflection coefficient [5].

We exploit this phenomenon to develop nonreciprocal devices such as isolators for use at millimeter wavelengths.

### III. 94-GHz REFLECTION-BEAM ISOLATOR

The experimental apparatus shown in Fig. 1 is designed to measure the reflection from the interface between a dielectric and solid-state plasma at room temperature. The semiconductor used for the experiment is n-type  $1.44 \Omega\cdot\text{cm}$  GaAs, with  $n = 7.1 \times 10^{14}/\text{cm}^3$ ,  $\mu = 6.1 \times 10^3 \text{ cm}^2/\text{V}\cdot\text{s}$ , and the lattice dielectric constant  $K_L = 12.53$ ,  $m^* = 0.067m_0$ .

A pyramidally cut dielectric (Stycast  $\epsilon = 15\epsilon_0$ , Emerson and Cuming) was used as a "lossless" dielectric medium, and the mirror-polished GaAs, 10 or more skin-depths thick under operating conditions, was mounted on the bottom of the pyramid as shown in Fig. 1. Quarter-wave matching plates (Stycast, Emerson and Cuming) were placed on the incident and transmitted sides. This device was placed in the 1-in air gap of a Varian 6-in magnet which provides a magnetic field up to 15 kG.

Pyramidal horns with a beam angle of  $7.6^\circ$  were used to provide the incident wave polarized in the plane of incidence and as a detector pickup. These horns were set about 10 cm away from the sample in order to ensure that the EM wave incident on the surface of the dielectric was reasonably planar. A lock-in amplifier was used to amplify and read the signal from the crystal detector. In order to eliminate the stray reflections and transmission, a window frame made out of lossy material (Eccosorb, Emerson and Cuming) was placed in front of the GaAs. This ensured that the detected signal was indeed the signal reflected from the GaAs, and was not due to stray transmission or reflection aside from GaAs.

The reflected power was measured as a function of magnetic field for incident angles  $\theta$  between  $30^\circ$  and  $75^\circ$ . Insertion loss due to the isolator was measured by replacing the GaAs with a plate of polished brass and comparing the reflected power.

### IV. THEORETICAL AND EXPERIMENTAL RESULTS

Theoretical reflection loss from GaAs with a semi-infinite dielectric ( $K_M = 15$ ) is compared to the experiment with the geometry of Fig. 1 and is shown in Fig. 2 for a dc magnetic field of 13.2 kG as a function of incident angle, and in Fig. 3 for an incident angle of  $57.5^\circ$  as a function of a dc magnetic field.

Isolation of 12 dB with 11-dB insertion loss was observed for the signal at an incident angle of  $57.5^\circ$ , whereas the theory predicts 44-dB isolation with 5-dB insertion loss at an incident angle of  $65^\circ$ . The deviation between theoretical and experimental results will be discussed.

The 94-GHz signal from the pyramidal horns used for transmitting and receiving had an angular spread of  $2^\circ$ – $5^\circ$  at the sample, depending on the tilt of the sample to the horn, and therefore the reflection obtained from the experiments was an "average" value of reflection for an angular spread. The spread in incident angles would not only smear out the sharpness of the reflection curve, but also reduce its peak predicted from the theory (see Fig. 2).

The size of the face of the dielectric pyramid was only about  $5 \times 7$  wavelengths, and the non-plane-wave behavior due to diffraction caused by this "small" window washed out the sharpness of the reflection loss. This was confirmed from the fact that a smaller window, 3 wavelengths square, destroyed the nonreciprocal phenomena.

Unlike the conditions reported earlier [4], the sample thickness (4.4 mm) was many skin depths; thus reflection from the backside of the GaAs was unimportant.

The parameters of GaAs were determined by the van der Pauw

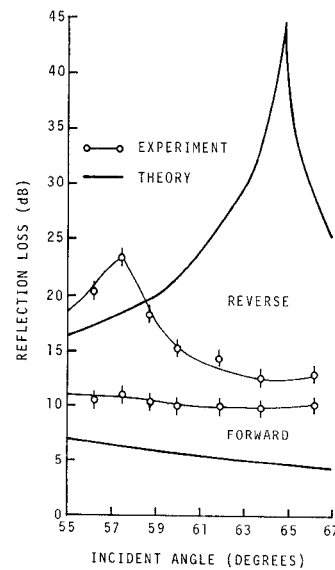


Fig. 2. Theoretical and experimental reflection loss of GaAs at 94 GHz as a function of incident angle. (Geometry is shown in Fig. 1 with  $B = 13.2$  kG.)

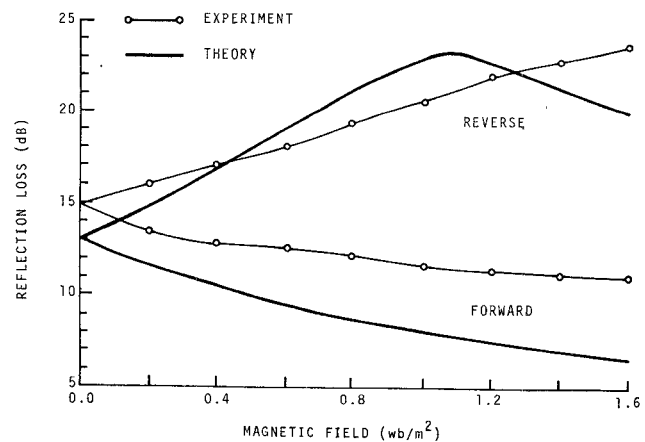


Fig. 3. Theoretical and experimental reflection loss of GaAs at 94 GHz as a function of magnetic field. (Geometry is shown in Fig. 1 with  $\theta = 57.5^\circ$ .)

measurement, a technique which gives an overall average value of the mobility and the carrier concentration of free carriers for the whole sample of GaAs. However, the measurements made on various pieces of material indicated that the GaAs used in our experiment exhibited variations of 10 percent in these parameters, causing some loss of sharpness in the isolation peak.

Although the observed efficiency of the device as an isolator was not very good due to the geometrical effects, the theory predicts that good performance should be possible.

### V. SUMMARY AND CONCLUSIONS

The theoretical and experimental investigations described were performed for the purpose of demonstrating a reflection-beam isolator at millimeter and submillimeter wavelengths using nonreciprocal reflection of EM waves incident on a semiconductor. For our experiments at 94 GHz, theory predicts over 40-dB isolation with 5-dB insertion loss. However, because of the sensitivity of the reflection loss on material parameters and geometry, the observed isolation was only 12 dB with 11-dB insertion loss. This structure has more promise at shorter wavelengths [5].

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### Numerical Analysis of Eigenvalue Solution of Disk Resonator

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**Abstract**—A formulation is proposed to calculate the frequencies of the eigenmodes for a resonator with a thin conductor disk placed in the median plane between two infinite parallel conductor plates. The numerical analysis is carried out for the  $E$ - and  $EH$ -modes, and these eigenvalues are calculated as the function of the ratio of the disk radius to the distance between the disk and one of the infinite conductor plates. It is shown that at a ratio greater than a certain value the exact eigenvalue is smaller than the one predicted by applying the conventional method for two-dimensional bifurcation of rectangular waveguide, but the latter becomes closer to the exact one with increasing ratio. The availability of our exact eigenvalues is demonstrated in determining experimentally the dielectric constant of Teflon plate specimen by applying those values. Then the constancy of the measured dielectric constant is confirmed irrespective of the modes and the ratios.

#### I. INTRODUCTION

In the recent experiments by Kobayashi *et al.* [1], [2] it is shown that the high-precision measurement of the resonance frequencies of the modes excited in the resonator containing dielectric plates between two sufficiently large parallel plates and a thin conductor disk placed in the median plane (see Fig. 1.) provides a possible method to determine accurately the dielectric constant. Present analysis is motivated by the facts that this accuracy depends upon the correctness of the relation between the measured frequency and the dielectric constant, as well as that the disk resonator plays an important role particularly in the planar circuits [3].

We consider the resonant cavity immersed with a dielectric substance of permittivity  $\epsilon$  and permeability  $\mu$ , assumed later to be vacuum value, as shown in Fig. 1. For simplicity the thickness of the disk is neglected, and the conductivity of the plates and the disk is assumed to be infinite. The axially symmetric property of such device is conveniently described in the cylindrical coordinates whose origin is at the center of the disk and  $z$  axis perpendicular to the plates and the disk.

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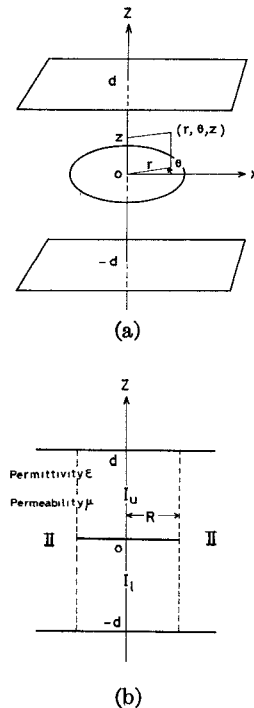


Fig. 1. Disk resonator. (a) General view. (b) Side view.

It is well known that as the ratio  $s = R/d$  for the disk radius  $R$  and the distance between the disk and one of the parallel plates become large, the eigenvalue of the  $E_{nm}$ -mode defined by  $x = kR$  with  $k = \omega(\epsilon\mu)^{1/2}$  approaches the  $m$ th root of the equation

$$J_n'(x_{nm}) = 0 \quad (1)$$

which is derived simply by requiring the boundary condition for an open circuit at disk edge. When  $s$  is not too small, the dependence on  $s$  must be taken into account by applying the conventional method for the two-dimensional bifurcation of a rectangular waveguide [4], which we refer to simply as the Marcuvitz method hereafter. Then the eigenvalue  $x_{nm}$  is approximated by the root of the equation given by

$$J_n'(x_{nm}') = 0 \quad (2)$$

with

$$x_{nm}' = \left(1 + \frac{2 \ln 2}{\pi s}\right) x_{nm} + S_1\left(\frac{2x_{nm}}{\pi s}; 0, 0\right) - 2S_1\left(\frac{x_{nm}}{\pi s}; 0, 0\right) \quad (3)$$

where the function  $S_1$  is defined by

$$S_1(z; 0, 0) = \sum_{n=1}^{\infty} [\sin^{-1}(z/n) - z/n]. \quad (4)$$

Obviously (2) reduces to (1) as  $s$  tends to infinity. When we neglect the second and third terms in (4), the solution of (2) corresponds to the one of (1) for the disk radius effectively enlarged by  $2 \ln 2/\pi$  [5], [6]. We refer to this approximation as the approximated Marcuvitz method.

#### II. EIGENVALUE FORMULATION

For a loss-free medium all the field components can be expressed in terms of the  $z$  components of Hertz vectors  $\Pi_e$  for electric mode and  $\Pi_m$  for magnetic mode satisfying the same Helmholtz equations

$$(\Delta + k^2)\Pi_e = 0 \quad \text{and} \quad (\Delta + k^2)\Pi_m = 0. \quad (5)$$

The most general solutions satisfying the appropriate boundary conditions are provided by